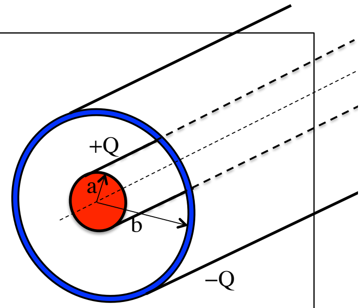
Problem 26.5

This is a great problem. The coaxial cable is shown to the right. We know that:

$$C = \frac{Q}{V},$$



so to derive our capacitance function we need to assume a charge "Q" on the inside rod and a charge "-Q" on the outside sheath, derive an expression for the voltage in the region between "a" and "b," and take the ratio of the charge to voltage. To do that, we need to use the fact that:

$$\Delta V = -\int_{r_a}^{r_b} \vec{E} \bullet d\vec{r}$$

Noticing that 1.) because we are moving from a positive plate to a negative plate, the voltage derived above will be *minus* the defined value for "the capacitor's voltage" (this is always positive, so $\Delta V = -V_C$), and 2.) that we can derive an expression for that electric field under the integral using Gauss's Law, we can write:

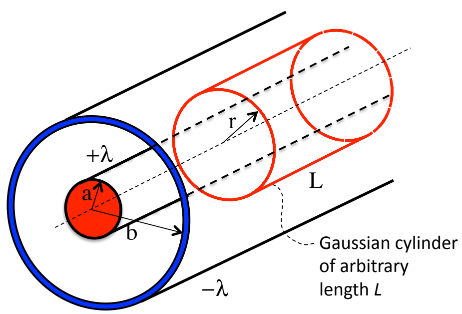
Using Gauss's Law:

$$\int \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi r L) = \frac{\lambda L}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\lambda L}{2\pi\varepsilon_o L r}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_o r}$$



So with the outside sheath being negative, we can take it to be grounded (zero voltage) and write:

$$V_{cap} = -\Delta V = +\int_a^b \vec{E} \bullet d\vec{r}$$

$$\Rightarrow \quad V_{cap} = \int_a^b \vec{E} \bullet d\vec{r}$$

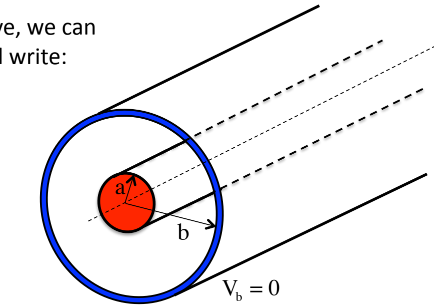
$$\Rightarrow \quad V_{cap} = \int_a^b \left(\frac{\lambda}{2\pi\varepsilon_o r}\right) dr \cos 0^o$$

$$\Rightarrow \quad V_{cap} = \frac{\lambda}{2\pi\varepsilon_o} \int_a^b \left(\frac{1}{r}\right) dr$$

$$\Rightarrow \quad V_{cap} = \frac{\lambda}{2\pi\varepsilon_o} \ln(r)\Big|_a^b$$

$$\Rightarrow \quad V_{cap} = \frac{\lambda}{2\pi\varepsilon_o}\left(\ln(b) - \ln(a)\right)$$

$$\Rightarrow \quad V_{cap} = \frac{\lambda}{2\pi\varepsilon_o}\left(\ln\left(\frac{b}{a}\right)\right)$$



If we remember that λ is *charge per unit length*, or *Q/L*, we can finish off the problem by writing:

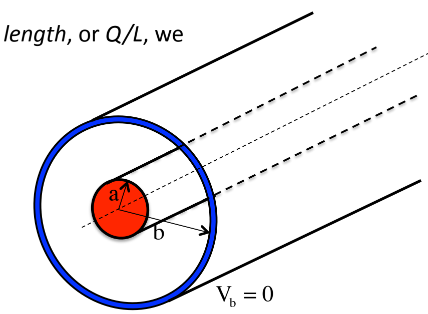
$$C = \frac{Q}{V_{cap}}$$

$$\Rightarrow \quad C = \frac{Q}{\left[\frac{\lambda}{2\pi\varepsilon_o}\left(\ln\left(\frac{b}{a}\right)\right)\right]}$$

$$\Rightarrow \quad C = \frac{Q}{\left[\frac{\left(Q/L\right)}{2\pi\varepsilon_o}\left(\ln\left(\frac{b}{a}\right)\right)\right]}$$

$$\Rightarrow \quad C = \frac{2\pi\varepsilon_o L}{\left(\ln\left(\frac{b}{a}\right)\right)}$$

$$\Rightarrow \quad C = \frac{L}{2k\left(\ln\left(\frac{b}{a}\right)\right)}$$

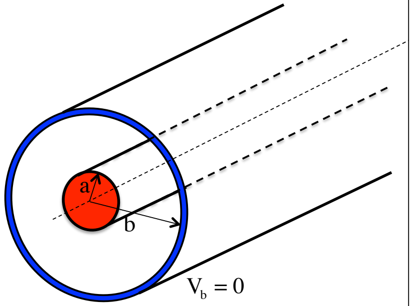


So for our problem, the numbers yield:

$$C = \frac{L}{2k\left(\ln\left(\frac{b}{a}\right)\right)}$$

$$= \frac{(50.0\text{ m})}{2\left(8.99\text{x}10^{9}\text{ N}\bullet\text{m}^2/\text{C}^2\right)\left(\ln\left(\frac{7.27\text{x}10^{-3}\text{ m}}{2.58\text{x}10^{-3}\text{ m}}\right)\right)}$$

$$= 2.68\text{x}10^{-9}\text{ F} \qquad (\text{or } 2.68\text{ nf})$$



b.) There are two ways to get the voltage difference between the plates.

Approach 1: As $C = \frac{Q}{V_C} \Rightarrow V_C = \frac{Q}{C}$

so

$$V_C = \frac{\left(8.10\text{x}10^{-6}\text{ C}\right)}{\left(2.68\text{x}10^{-9}\text{ F}\right)}$$

$$= 3.02\text{x}10^{3}\text{ V}$$

Approach 2: From the earlier derivation:

$$V_{cap} = \frac{\lambda}{2\pi\varepsilon_o}\left(\ln\left(\frac{b}{a}\right)\right)$$

$$\Rightarrow \quad V_{cap} = 2k\lambda\left(\ln\left(\frac{b}{a}\right)\right)\frac{\left(8.10\text{x}10^{-6}\text{ C}\right)}{\left(2.68\text{x}10^{-9}\text{ F}\right)}$$

$$\Rightarrow \quad V_{cap} = 2\left(8.99\text{x}10^{9}\text{ N}\bullet\text{m}^2/\text{C}^2\right)\left(\frac{8.10\text{x}10^{-6}\text{ C}}{50.0\text{ m}}\right)\left(\ln\left(\frac{7.27\text{x}10^{-3}\text{ m}}{2.58\text{x}10^{-3}\text{ m}}\right)\right)$$

$$= 3.02\text{x}10^{3}\text{ V}$$